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Financial portfolio optimization with quantum computing

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Agenda

- Portfolio optimization basics
- Quantum computing basics
- Putting it together
- Demo

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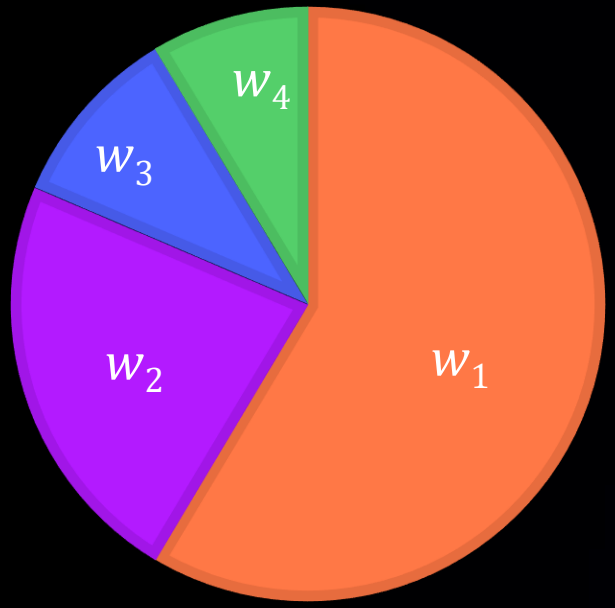
Portfolio optimization

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Modern portfolio theory: Markowitz model

Given: N investable assets

Find: the optimal allocation of funds into the assets to maximize expected returns and/or minimize risk







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Modern portfolio theory: Markowitz model

Minimize risk

$$\min_{\boldsymbol{w}} \quad \frac{1}{2}\boldsymbol{w}^{\top}\Sigma\boldsymbol{w}$$

with fixed return

Maximize return

$$\max_{\boldsymbol{w}} \quad \boldsymbol{w}^{\top}\boldsymbol{r}$$

with fixed risk

Optimize with risk-aversion parameter

$$\max_{\boldsymbol{w}} \quad \boldsymbol{w}^{\top}\boldsymbol{r} - \frac{1}{2}\gamma\boldsymbol{w}^{\top}\Sigma\boldsymbol{w}$$

with fixed budget

Definitions:

- $\boldsymbol{w}$—vector specifying fraction of total investment allocated to each asset
- $\boldsymbol{r}$—vector of returns; r_0 is a target expected return
- Σ—covariance matrix for the assets; σ_0^2 is a target variance
- γ—risk-aversion parameter

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Modern portfolio theory: Markowitz model

Minimize risk

$$\min_{\boldsymbol{w}} \quad \frac{1}{2}\boldsymbol{w}^{\top}\Sigma\boldsymbol{w}$$

Subject to

$$\boldsymbol{w}^{\top}\boldsymbol{r} = r_0$$

$$\boldsymbol{w}^{\top}\mathbf{1} = 1$$

Maximize return

$$\max_{\boldsymbol{w}} \quad \boldsymbol{w}^{\top}\boldsymbol{r}$$

Subject to

$$\boldsymbol{w}^{\top}\Sigma\mathbf{w} = \sigma_0^2$$

$$\boldsymbol{w}^{\top}\mathbf{1} = 1$$

Optimize with risk-aversion parameter

$$\max_{\boldsymbol{w}} \quad \boldsymbol{w}^{\top}\boldsymbol{r} - \frac{1}{2}\gamma\boldsymbol{w}^{\top}\Sigma\boldsymbol{w}$$

Subject to

$$\boldsymbol{w}^{\top}\mathbf{1} = 1$$

Definitions:

- $\boldsymbol{w}$—vector specifying fraction of total investment allocated to each asset
- $\boldsymbol{r}$—vector of returns; r_0 is a target expected return
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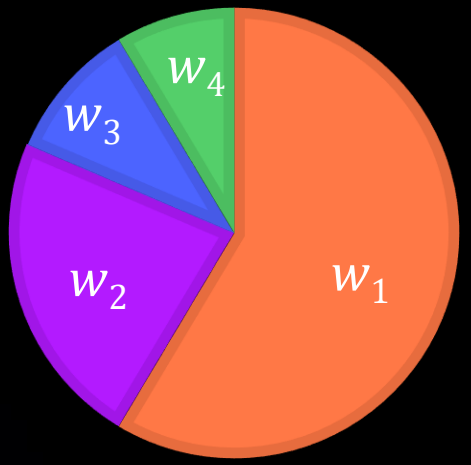
Modern portfolio theory: Constraints

Constraints can make the problem richer and more complex:

$$\max_{\boldsymbol{w}} \boldsymbol{w}^\top \boldsymbol{r} - \frac{1}{2}\gamma \boldsymbol{w}^\top \Sigma \boldsymbol{w}$$

Subject to

$$\boldsymbol{w}^\top \mathbf{1} = 1$$



- $w_j \geq 0, \forall\, j$ Long only (i.e., no short trades)
- $w_j \in [w_j^{\min}, w_j^{\max}]$ Investment bands with minimum and maximum holding for asset j
- $\left|\Delta w_j\right| \leq U_j$ Turnover constraints on asset j
- Many other possibilities: sector constraints, cardinality constraints, transaction costs, etc.

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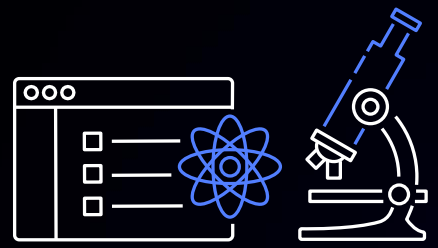
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Quantum computing

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What is quantum computing?

- A new model of computation that is different from all others
- A way of processing information using the laws of quantum physics
  - The information content itself can be different: quantum information
  - Leverages features of quantum systems like superposition and entanglement
- A paradigm for computing on specialized hardware
- An area of active research with countless open problems



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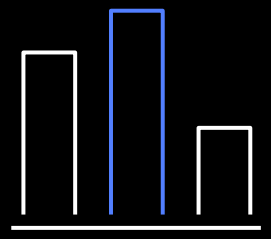
Quantum solutions will be widely applicable

IN THE FULLNESS OF TIME

Financial services
Automotive
Energy and gas

Pharmaceutical
Biotechnology
Manufacturing

Academic research
Government
...



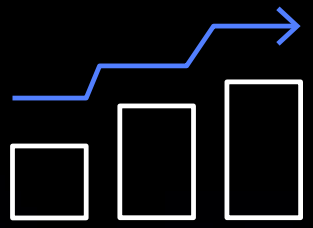
Optimization



Chemistry and materials



Machine learning



Financial services

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QUBO problems can be solved near-term

Quadratic Unconstrained Binary Optimization (QUBO)

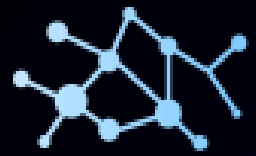
$$\text{minimize/maximize } Y = \sum_{i,j} Q_{i,j} X_i X_j = X^\top Q X, \text{ where } X_i \in 0, 1 \text{ and } Q_{i,j} \in \mathbb{R}$$

AWS enables a multi-pronged approach to solving QUBO problems

Quantum native

D-Wave
The Quantum Computing Company™

Physics inspired



Meta-Analytics

Open-source and commercial solvers



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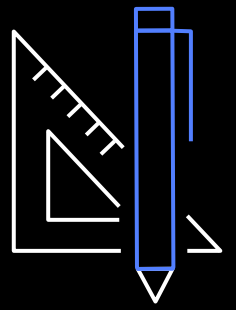
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The Amazon Quantum Solutions Lab's offering

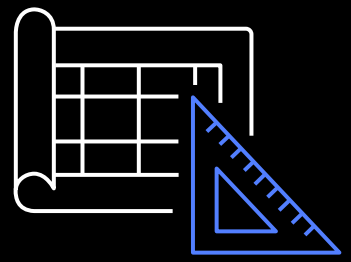
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How to optimize a portfolio

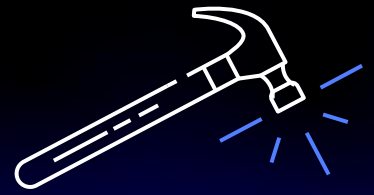
- **Step 1:** Formulate the problem (including constraints)



- **Step 2:** Map to the right representation for the backend



- **Step 3:** Solve the problem on the chosen backend



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Optimize portfolios with quantum computers

- E.g., form Hamiltonian: $H = -\boldsymbol{w}^\top \boldsymbol{r} + \frac{1}{2}\gamma \boldsymbol{w}^\top \Sigma \boldsymbol{w} + \rho(\boldsymbol{w}^\top \mathbf{1} - 1)^2 + \cdots$
- Map to QUBO (introduces overhead):
  - Binary representation of weights w_j
  - Use slack variables and/or other techniques to implement constraints
- For quantum hardware: QUBO is embedded into underlying graph
- Solve using backend of choice

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Thank you!

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